**PSEUDO SPECTRAL METHOD FOR THE SOLUTION OF BOUNDARY VALUE PROBLEM**

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***ABSTRACT:*** *The aim of this work is to find the solution of boundary value problems. We use the pseudo-spectral method to solve the boundary value problem as this method is best for approximate solution. We solve an example which shows that Pseudo-Spectral method has a finite rate of convergence and has little errors than finite difference method.*

**Key Words:** Pseudo-Spectral Method, Chebyshev Polynomial, Boundary value problem.

**1.1 INTRODUCTION:**

Numerical methods are classified on the basis of their global or local nature. Efficiency of local approach is enhanced by reducing the typical mash length. Efficiency of global approach is increased by increasing the order of the polynomials. The drawback being, that they are difficult to use when applied on the complex geometries. Finite element methods find their own way out. They are elastic to use and hence are used in structural mechanics. Spectral methods represent the solution of differential equations. It includes a truncated set of global approach from an ontological set. The scheme is, of course, most effective for problem with smooth solution, in which case they converge faster than any finite power of typical mesh size. This property is known as exponential convergence. We presented some examples containing corner singularities indicating that although the spectral method had a finite rate of convergence it had a lower absolute error than the finite difference method. Spectral methods have evolved quite a lot in the last fifteen years. These methods are now an established alternative to finite difference and finite element method for solving differential equations. There is an extensive literature on the subject see [1], but we review some of the research performed in this area. The multi-domain spectral method of Morchoisne has been used in computational fluid dynamics to solve for the flow past an aerofoil. This procedure does not use a variation formulation or trail functions that are continuous across the artificial boundaries. The continuity of the approximation to the solution and its normal derivative across the boundaries is obtained by an iterative procedure.

They emerged in the last fifteen years. They provide an alternative to finite difference as well as finite element methods. They are very efficient for solving differential equation. A large amount of literature is available on this subject. Multi domain of spectral approach is widely used in fluid dynamics. Equations have been solved with ease regarding dynamics. No special formula or trial functions are involved. Global domain methods have been extensively used in the solution of boundary value problems as well as in computational dynamics. A method has been introduced by Delves [10] for solving elliptic problems. In this method, it is not necessary to satisfy the boundary functions. It is being imposed by a variation problem. The spectral element method of Phillips and Davies [2] describe poison problems defined in irregular infinite domains to be solved as a set of coupled problems over semi-infinite rectangular regions. Two choices of trail functions are considered, namely the Eigen-function of the differential operator and chebyshev polynomials. The coefficients in the series expansion are obtained by imposing weak matching conditions across element interface. The work and develop method for more complicated domains, for example, T-shaped geometry. The chebyshev Collection method of Phillips and Karageorhis [3] solve the problem of incompressible Navier-stokes equation in complex geometries. Same authors Karageorhis and Phillips [7] describe chebyshev spectral collection methods for Laminar flow through a channel contraction. Then Karageorhis and Phillips [8] describe Spectral Collection methods for the stoke flow in contraction geometry and unbounded domains. Phillips [4] developed a spectral tau-method, which results in a well-conditioned matrix problem for the unknown Chebyshev expansion coefficient in a spectral approximation to the solution of fourth order problem. The condition number grow like O() instead of O() obtained as a result of standard implementation of spectral method. Method devised by Greenstones is known as cell Discretization method. It has interface conditions which are in the form of explicit, exact constrains. For more detail of spectral method and boundary value problems see [5, 6, 9, 10, 11]. In this paper we find out the solution of boundary value problems. For this purpose we use pseudo-spectral method to solve the boundary value problem as this method is best for approximate solution. We solve a problem which shows that Pseudo-Spectral method has a finite rate of convergence and has little errors than finite difference method.

**1.2 NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATION**

Three techniques mentioned below for solution of differential equation

1. Finite Difference Techniques
2. Finite Element Methods
3. Spectral Method

We use the Spectral method to find solution.

**1.3 PSEUDO-SPECTRAL METHOD**

Spectral methods were produced in the extended group of reports by Steven Orszag commencing in 1969 including, however, not on a, Fourier line options for regular geometry difficulties, polynomial spectral options for limited along with unbounded geometry difficulties, pseudo spectral options for highly nonlinear difficulties, along with spectral technology options for quick option regarding regular point out difficulties. The implementation in the spectral method is often done either having collocation or a Gale kin or a Tau approach. Spectral methods usually are computationally more affordable as compared to limited element methods; yet turn out to be a smaller amount exact for issues with sophisticated geometries along with discontinuous coefficients. This particular raise in problem is a consequence of the actual Gibbs phenomenon.

In this paper, we are concerned with solving elliptic problems inside rectangular decomposable domains. It is seen that by using the spectral method, that whole domain is changed by even a slight change in the trail solution. For solving the elliptic, hyperbolic or parabolic problems, spectral method is best suited for it. Spectral method’s efficiency and accuracy is dependent on choosing the trial functions.

**1.4 FUNDAMENTALS OF SPECTRAL METHOD**

Consider an equation

, ( is linear differential operator.)

,

bounded domain, boundary

is known as the second order linear differential operator. is the boundary operator whereas the solution is

Expression of form for the spectral method is

The trail function selected as first terms of orthogonal system as . The unknown co-efficient being the . Test functions are satisfied by the expansion of truncated series. It can be achieved by reducing the residual that being the error in the differential equation.

The two methods; Galerkin and Collocation are different from each other by the spectral schemes. Details can be taken from Gottleib and Orszag and Canuto et al..

Spectral method’s efficiency and accuracy is dependent on choosing the trial functions.

**1.5 CHOICE OF TRAIL FUNCTION**

It is also possible to show the power of in form of Chebyshev polynomials

, , ,

And so on. These expressions will be useful for economization of power series to be discussed later. For domain we consider the interval .

**1.6 PROBLEM**

We suggest for the solution of the equation.

With and

The boundary condition, it is simply asked that the solution is zero at the boundaries.

Under this condition, the solution is unique and analytical

**Solution:** ForN = 4

by substituting the value of chebyshev polynomial of 2nd kind

Equation (i) becomes

For

For

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | -2 | 3 | -4 | 5 | a0 |  | 0 |
| 1 | 2.382 | -3.326076 | -16.41671303 | 72.71928956 | a1 |  | 0.3074 |
| 1 | 4.618 | 12.325924 | 10.41511703 | -16.72308955 | a2 | = | 1.2976 |
| 1 | 5.618 | 22.561924 | 63.24688903 | 130.6009226 | a3 |  | 1.5500 |
| 1 | 2 | 3 | 4 | 5 | a4 |  | 0 |

Table for

|  |  |  |  |
| --- | --- | --- | --- |
| Points | Exact solution | Numerical solution | Error |
|  | 0 | 0 | 0 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  | 0 | 0 | 0 |

For N = 8

|  |  |  |  |
| --- | --- | --- | --- |
| Points | Exact solution | Numerical solution | Error |
|  | 0 | 0 | 0 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  | 0 | 0 | 0 |

For N = 16

|  |  |  |  |
| --- | --- | --- | --- |
| Points | Exact solution | Numerical solution | Error |
|  | 0 | 0 | 0 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  | 0 | 0 | 0 |

**CONCLUSION:**

The Pseudo-spectral Method is used to solve linear second order boundary value problems for ODE. We also studied the accuracy of the developed scheme. Problem is solved numerically by using Pseudo-spectral Method. The numerical results are compared with exact solution.

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